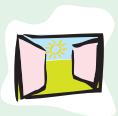
Chapter 3

Algebraic expansion and simplification

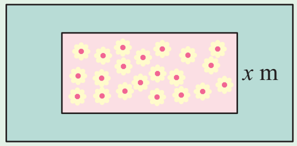
Contents:

The study of **algebra** is an important part of the problem solving process. When we convert real life problems into algebraic equations, we often obtain expressions that need to be **expanded** and **simplified**.

OPENING PROBLEM



Ethel is planning a rectangular flower bed with a lawn of constant width around it. The lawn's outer boundary is also rectangular. The shorter side of the flower bed is x m long.



- If the flower bed's length is 4 m longer than its width, what is its width?
- If the width of the lawn's outer boundary is double the width of the flower bed, what are the dimensions of the flower bed?
- Wooden strips form the boundaries of the flower bed and lawn. Find, in terms of x, the total length L of wood required.

A COLLECTING LIKE TERMS

In algebra,

> **like terms** are terms which contain the same variables (or letters) to the same indices.

For example:

- xy and $-2xy$ are **like terms**.
- x^2 and $3x$ are **unlike terms** because the powers of x are not the same.

Algebraic expressions can often be simplified by adding or subtracting like terms. We call this **collecting like terms**.

Consider $2a + 3a = \underbrace{a + a}_{\text{2 lots of } a} + \underbrace{a + a + a}_{\text{3 lots of } a} = \underbrace{5a}_{\text{5 lots of } a}$

Example 1 Self Tutor

Where possible, simplify by collecting the terms:

a $4x + 3x$ **b** $5y - 2y$ **c** $2a - 1 + a$ **d** $mn - 2mn$ **e** $a^2 - 4a$

a $4x + 3x = 7x$

b $5y - 2y = 3y$

c $2a - 1 + a = 3a - 1$ {since $2a$ and a are like terms}

d $mn - 2mn = -mn$ {since mn and $-2mn$ are like terms}

e $a^2 - 4a$ cannot be simplified since a^2 and $-4a$ are unlike terms.

Example 2 — Self Tutor

Simplify by collecting like terms:

a $-a-1+3a+4$ **b** $5a-b^2+2a-3b^2$

a $-a-1+3a+4$
$=-a+3a-1+4$
$=2a+3$
{$-a$ and $3a$ are like terms
-1 and 4 are like terms}

b $5a-b^2+2a-3b^2$
$=5a+2a-b^2-3b^2$
$=7a-4b^2$
{$5a$ and $2a$ are like terms
$-b^2$ and $-3b^2$ are like terms}

EXERCISE 3A

1 Simplify, where possible, by collecting like terms:

a $5+a+4$ **b** $6+3+a$ **c** $m-2+5$ **d** $x+1+x$
e $f+f-3$ **f** $5a+a$ **g** $5a-a$ **h** $a-5a$
i x^2+2x **j** d^2+d^2+d **k** $5g+5$ **l** x^2-5x^2+5
m $2a+3a-5$ **n** $2a+3a-a$ **o** $4xy+xy$ **p** $3x^2z-x^2z$

2 Simplify, where possible:

a $7a-7a$ **b** $7a-a$ **c** $7a-7$
d $xy+2yx$ **e** $cd-2cd$ **f** $4p^2-p^2$
g $x+3+2x+4$ **h** $2+a+3a-4$ **i** $2y-x+3y+3x$
j $3m^2+2m-m^2-m$ **k** $ab+4-3+2ab$ **l** x^2+2x-x^2-5
m $x^2+5x+2x^2-3x$ **n** $ab+b+a+4$ **o** $2x^2-3x-x^2-7x$

3 Simplify, where possible:

a $4x+6-x-2$ **b** $2c+d-2cd$ **c** $3ab-2ab+ba$
d $x^2+2x^2+2x^2-5$ **e** p^2-6+2p^2-1 **f** $3a+7-2a-10$
g $-3a+2b-a-b$ **h** a^2+2a-a^3 **i** $2a^2-a^3-a^2+2a^3$
j $4xy-x-y$ **k** $xy^2+x^2y+x^2y$ **l** $4x^3-2x^2-x^3-x^2$

B PRODUCT NOTATION

In algebra we agree:

- to **leave out** the "$\times$" **signs** between any multiplied quantities provided that at least one of them is an unknown (letter)
- to write **numerals (numbers) first** in any product
- where products contain two or more letters, we write them in **alphabetical order**.

For example:
- $2a$ is used rather than $2\times a$ or $a2$
- $2ab$ is used rather than $2ba$.

ALGEBRAIC PRODUCTS

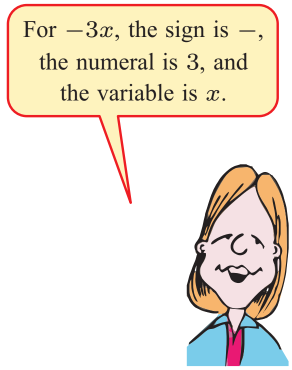
The **product** of two or more factors is the result obtained by multiplying them together.

Consider the factors $-3x$ and $2x^2$. Their product $-3x \times 2x^2$ can be simplified by following the steps below:

Step 1: Find the product of the **signs**.

Step 2: Find the product of the **numerals** or numbers.

Step 3: Find the product of the **variables** or letters.



So, $$-3x \times 2x^2 = -6x^3$$

$- \times + = -$

$3 \times 2 = 6$

$x \times x^2 = x^3$

Example 3 **Self Tutor**

Simplify the following products:

a $-3 \times 4x$ **b** $2x \times -x^2$ **c** $-4x \times -2x^2$

a $-3 \times 4x$
$= -12x$

b $2x \times -x^2$
$= -2x^3$

c $-4x \times -2x^2$
$= 8x^3$

EXERCISE 3B

1 Write the following algebraic products in simplest form:

a $c \times b$ **b** $a \times 2 \times b$ **c** $y \times xy$ **d** $pq \times 2q$

2 Simplify the following:

a $2 \times 3x$ **b** $4x \times 5$ **c** $-2 \times 7x$ **d** $3 \times -2x$

e $2x \times x$ **f** $3x \times 2x$ **g** $-2x \times x$ **h** $-3x \times 4$

i $-2x \times -x$ **j** $-3x \times x^2$ **k** $-x^2 \times -2x$ **l** $3d \times -2d$

m $(-a)^2$ **n** $(-2a)^2$ **o** $2a^2 \times a^2$ **p** $a^2 \times -3a$

3 Simplify the following:

a $2 \times 5x + 3x \times 4$ **b** $5 \times 3x - 2y \times y$ **c** $3 \times x^2 + 2x \times 4x$

d $a \times 2b + b \times 3a$ **e** $4 \times x^2 - 3x \times x$ **f** $3x \times y - 2x \times 2y$

g $3a \times b + 2a \times 2b$ **h** $4c \times d - 3c \times 2d$ **i** $3a \times b - 2c \times a$

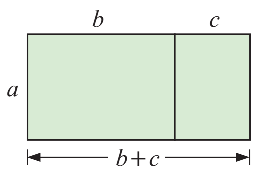
C THE DISTRIBUTIVE LAW

Consider the expression $2(x+3)$. We say that 2 is the **coefficient** of the expression in the brackets. We can **expand** the brackets using the **distributive law**:

$$a(b+c) = ab + ac$$

The distributive law says that we must multiply the coefficient by each term within the brackets, and add the results.

Geometric Demonstration:



The overall area is $a(b+c)$.

However, this could also be found by adding the areas of the two small rectangles, i.e., $ab + ac$.

So, $a(b+c) = ab + ac$. {equating areas}

Example 4 Self Tutor

Expand the following:

a $3(4x+1)$ **b** $2x(5-2x)$ **c** $-2x(x-3)$

a $3(4x+1)$
$= 3 \times 4x + 3 \times 1$
$= 12x + 3$

b $2x(5-2x)$
$= 2x(5+-2x)$
$= 2x \times 5 + 2x \times -2x$
$= 10x - 4x^2$

c $-2x(x-3)$
$= -2x(x+-3)$
$= -2x \times x + -2x \times -3$
$= -2x^2 + 6x$

With practice, we do not need to write all of these steps.

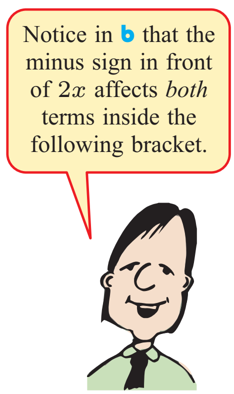
Example 5 Self Tutor

Expand and simplify:

a $2(3x-1) + 3(5-x)$ **b** $x(2x-1) - 2x(5-x)$

a $2(3x-1) + 3(5-x)$
$= 6x - 2 + 15 - 3x$
$= 3x + 13$

b $x(2x-1) - 2x(5-x)$
$= 2x^2 - x - 10x + 2x^2$
$= 4x^2 - 11x$



EXERCISE 3C

1 Expand and simplify:

a $3(x+1)$	**b** $2(5-x)$	**c** $-(x+2)$	**d** $-(3-x)$
e $4(a+2b)$	**f** $3(2x+y)$	**g** $5(x-y)$	**h** $6(-x^2+y^2)$
i $-2(x+4)$	**j** $-3(2x-1)$	**k** $x(x+3)$	**l** $2x(x-5)$
m $-3(x+2)$	**n** $-4(x-3)$	**o** $-(3-x)$	**p** $-2(x-y)$
q $a(a+b)$	**r** $-a(a-b)$	**s** $x(2x-1)$	**t** $2x(x^2-x-2)$

2 Expand and simplify:

a $1+2(x+2)$	**b** $13-4(x+3)$	**c** $3(x-2)+5$
d $4(3-x)-10$	**e** $x(x-1)+x$	**f** $2x(3-x)+x^2$
g $2a(b-a)+3a^2$	**h** $4x-3x(x-1)$	**i** $7x^2-5x(x+2)$

3 Expand and simplify:

a $3(x-4)+2(5+x)$	**b** $2a+(a-2b)$	**c** $2a-(a-2b)$
d $3(y+1)+6(2-y)$	**e** $2(y-3)-4(2y+1)$	**f** $3x-4(2-3x)$
g $2(b-a)+3(a+b)$	**h** $x(x+4)+2(x-3)$	**i** $x(x+4)-2(x-3)$
j $x^2+x(x-1)$	**k** $-x^2-x(x-2)$	**l** $x(x+y)-y(x+y)$
m $-4(x-2)-(3-x)$	**n** $5(2x-1)-(2x+3)$	**o** $4x(x-3)-2x(5-x)$

D THE PRODUCT $(a+b)(c+d)$

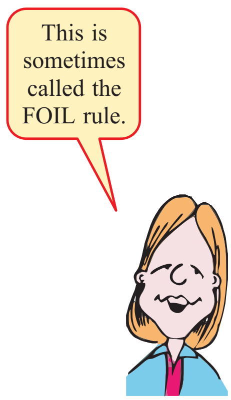
Consider the product $(a+b)(c+d)$.

It has two **factors**, $(a+b)$ and $(c+d)$.

We can evaluate this product by using the distributive law several times.

$$\begin{aligned}(a+b)(c+d) &= a(c+d)+b(c+d)\\ &= ac+ad+bc+bd\end{aligned}$$

So, $\quad \boldsymbol{(a+b)(c+d) = ac+ad+bc+bd}$



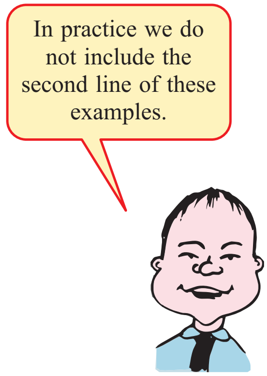
The final result contains four terms:

ac is the product of the **F**irst terms of each bracket.
ad is the product of the **O**uter terms of each bracket.
bc is the product of the **I**nner terms of each bracket.
bd is the product of the **L**ast terms of each bracket.

Example 6 — Self Tutor

Expand and simplify: $(x+3)(x+2)$.

$$(x+3)(x+2)$$
$$= x \times x + x \times 2 + 3 \times x + 3 \times 2$$
$$= x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

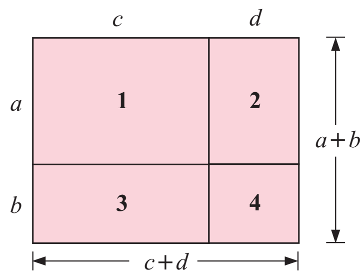


Example 7 — Self Tutor

Expand and simplify: $(2x+1)(3x-2)$

$$(2x+1)(3x-2)$$
$$= 2x \times 3x + 2x \times -2 + 1 \times 3x + 1 \times -2$$
$$= 6x^2 - 4x + 3x - 2$$
$$= 6x^2 - x - 2$$

EXERCISE 3D



1 Consider the figure alongside:
Give an expression for the area of:

- **a** rectangle 1
- **b** rectangle 2
- **c** rectangle 3
- **d** rectangle 4
- **e** the overall rectangle.

What can you conclude?

2 Use the rule $(a+b)(c+d) = ac + ad + bc + bd$ to expand and simplify:

- **a** $(x+3)(x+7)$
- **b** $(x+5)(x-4)$
- **c** $(x-3)(x+6)$
- **d** $(x+2)(x-2)$
- **e** $(x-8)(x+3)$
- **f** $(2x+1)(3x+4)$
- **g** $(1-2x)(4x+1)$
- **h** $(4-x)(2x+3)$
- **i** $(3x-2)(1+2x)$
- **j** $(5-3x)(5+x)$
- **k** $(7-x)(4x+1)$
- **l** $(5x+2)(5x+2)$

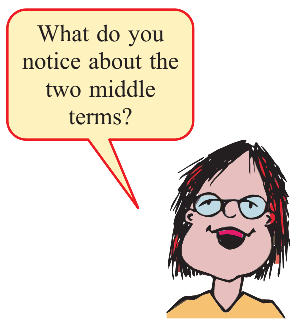
Example 8 — Self Tutor

Expand and simplify:

a $(x+3)(x-3)$ **b** $(3x-5)(3x+5)$

a $(x+3)(x-3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$

b $(3x-5)(3x+5)$
$= 9x^2 + 15x - 15x - 25$
$= 9x^2 - 25$



3 Expand and simplify:

a $(x+2)(x-2)$ **b** $(a-5)(a+5)$ **c** $(4+x)(4-x)$

d $(2x+1)(2x-1)$ **e** $(5a+3)(5a-3)$ **f** $(4+3a)(4-3a)$

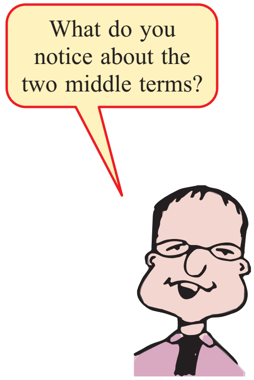
Example 9 Self Tutor

Expand and simplify:

a $(3x+1)^2$ **b** $(2x-3)^2$

a $(3x+1)^2$
$= (3x+1)(3x+1)$
$= 9x^2+3x+3x+1$
$= 9x^2+6x+1$

b $(2x-3)^2$
$= (2x-3)(2x-3)$
$= 4x^2-6x-6x+9$
$= 4x^2-12x+9$



4 Expand and simplify:

a $(x+3)^2$ **b** $(x-2)^2$ **c** $(3x-2)^2$

d $(1-3x)^2$ **e** $(3-4x)^2$ **f** $(5x-y)^2$

E DIFFERENCE OF TWO SQUARES

a^2 and b^2 are perfect squares and so a^2-b^2 is called the **difference of two squares**.

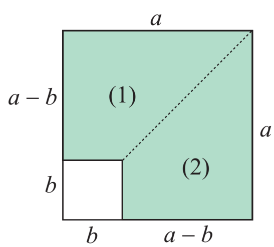
Notice that $(a+b)(a-b) = a^2 \underbrace{-ab+ab}_{} - b^2 = a^2-b^2$

the middle two terms add to zero

Thus, $$(a+b)(a-b) = a^2-b^2$$

Geometric Demonstration:

Consider the figure alongside:

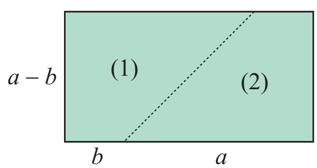


The shaded area

$=$ area of large square $-$ area of small square

$= a^2-b^2$

Cutting along the dotted line and flipping (2) over, we can form a rectangle.



The rectangle's area is $(a+b)(a-b)$.

$\therefore$ $(a+b)(a-b) = a^2-b^2$

COMPUTER DEMO

Example 10 Self Tutor

Expand and simplify:

a $(x+5)(x-5)$ **b** $(3-y)(3+y)$

a $(x+5)(x-5)$
$= x^2 - 5^2$
$= x^2 - 25$

b $(3-y)(3+y)$
$= 3^2 - y^2$
$= 9 - y^2$

Example 11 Self Tutor

Expand and simplify:

a $(2x-3)(2x+3)$ **b** $(5-3y)(5+3y)$

a $(2x-3)(2x+3)$
$= (2x)^2 - 3^2$
$= 4x^2 - 9$

b $(5-3y)(5+3y)$
$= 5^2 - (3y)^2$
$= 25 - 9y^2$

Example 12 Self Tutor

Expand and simplify: $(3x+4y)(3x-4y)$

$(3x+4y)(3x-4y)$
$= (3x)^2 - (4y)^2$
$= 9x^2 - 16y^2$

EXERCISE 3E

1 Expand and simplify using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(x+2)(x-2)$ **b** $(x-2)(x+2)$ **c** $(2+x)(2-x)$
d $(2-x)(2+x)$ **e** $(x+1)(x-1)$ **f** $(1-x)(1+x)$
g $(x+7)(x-7)$ **h** $(c+8)(c-8)$ **i** $(d-5)(d+5)$
j $(x+y)(x-y)$ **k** $(4+d)(4-d)$ **l** $(5+e)(5-e)$

2 Expand and simplify using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(2x-1)(2x+1)$ **b** $(3x+2)(3x-2)$ **c** $(4y-5)(4y+5)$
d $(2y+5)(2y-5)$ **e** $(3x+1)(3x-1)$ **f** $(1-3x)(1+3x)$
g $(2-5y)(2+5y)$ **h** $(3+4a)(3-4a)$ **i** $(4+3a)(4-3a)$

3 Expand and simplify using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(2a+b)(2a-b)$ **b** $(a-2b)(a+2b)$ **c** $(4x+y)(4x-y)$
d $(4x+5y)(4x-5y)$ **e** $(2x+3y)(2x-3y)$ **f** $(7x-2y)(7x+2y)$

INVESTIGATION THE PRODUCT OF THREE CONSECUTIVE INTEGERS

Con was trying to multiply $19 \times 20 \times 21$ without a calculator. Aimee told him to 'cube the middle integer and then subtract the middle integer' to get the answer.

What to do:

1 Find $19 \times 20 \times 21$ using a calculator.

2 Find $20^3 - 20$ using a calculator. Does Aimee's rule seem to work?

3 Check that Aimee's rule works for the following products:

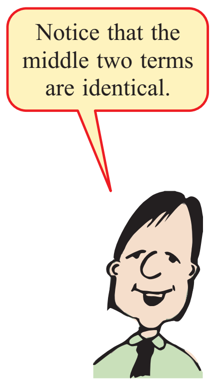
a $4 \times 5 \times 6$ **b** $9 \times 10 \times 11$ **c** $49 \times 50 \times 51$

4 Let the middle integer be x, so the other integers must be $(x-1)$ and $(x+1)$. Find the product $(x-1) \times x \times (x+1)$ by expanding and simplifying. Have you proved Aimee's rule?
Hint: Use the difference between two squares expansion.

F PERFECT SQUARES EXPANSION

$(a+b)^2$ and $(a-b)^2$ are called **perfect squares**.

Notice that
$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ab + b^2 \quad \{\text{using 'FOIL'}\}\\ &= a^2 + 2ab + b^2\end{aligned}$$



Thus, we can state the perfect square expansion rule:

$$\boldsymbol{(a+b)^2 = a^2 + 2ab + b^2}$$

We can remember the rule as follows:

Step 1: Square the *first term*.

Step 2: Add twice the product of the *first* and *last terms*.

Step 3: Add on the square of the *last term*.

Notice that
$$\begin{aligned}(a-b)^2 &= (a+(-b))^2\\ &= a^2 + 2a(-b) + (-b)^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Once again, we have the square of the first term, twice the product of the first and last terms, and the square of the last term.

Example 13 Self Tutor

Expand and simplify:

a $(x+3)^2$ **b** $(x-5)^2$

a $(x+3)^2$
$= x^2 + 2 \times x \times 3 + 3^2$
$= x^2 + 6x + 9$

b $(x-5)^2$
$= (x + -5)^2$
$= x^2 + 2 \times x \times (-5) + (-5)^2$
$= x^2 - 10x + 25$

Example 14 Self Tutor

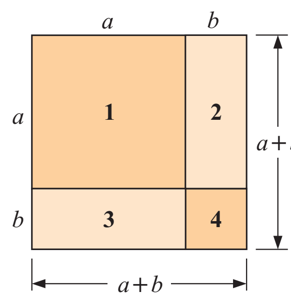
Expand and simplify using the perfect square expansion rule:

a $(5x+1)^2$ **b** $(4-3x)^2$

a $(5x+1)^2$
$= (5x)^2 + 2 \times 5x \times 1 + 1^2$
$= 25x^2 + 10x + 1$

b $(4-3x)^2$
$= (4 + -3x)^2$
$= 4^2 + 2 \times 4 \times (-3x) + (-3x)^2$
$= 16 - 24x + 9x^2$

EXERCISE 3F



1 Consider the figure alongside:
Give an expression for the area of:
a square 1 **b** rectangle 2 **c** rectangle 3
d square 4 **e** the overall square.
What can you conclude?

2 Use the rule $(a+b)^2 = a^2 + 2ab + b^2$ to expand and simplify:
a $(x+5)^2$ **b** $(x+4)^2$ **c** $(x+7)^2$
d $(a+2)^2$ **e** $(3+c)^2$ **f** $(5+x)^2$

3 Expand and simplify using the perfect square expansion rule:
a $(x-3)^2$ **b** $(x-2)^2$ **c** $(y-8)^2$
d $(a-7)^2$ **e** $(5-x)^2$ **f** $(4-y)^2$

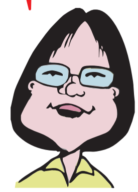
4 Expand and simplify using the perfect square expansion rule:
a $(3x+4)^2$ **b** $(2a-3)^2$ **c** $(3y+1)^2$
d $(2x-5)^2$ **e** $(3y-5)^2$ **f** $(7+2a)^2$
g $(1+5x)^2$ **h** $(7-3y)^2$ **i** $(3+4a)^2$

Example 15 **Self Tutor**

Expand and simplify: **a** $(2x^2+3)^2$ **b** $5-(x+2)^2$

a $(2x^2+3)^2$
$= (2x^2)^2 + 2\times 2x^2 \times 3 + 3^2$
$= 4x^4 + 12x^2 + 9$

b $5-(x+2)^2$
$= 5 - [x^2+4x+4]$
$= 5 - x^2 - 4x - 4$
$= 1 - x^2 - 4x$



5 Expand and simplify:

a $(x^2+2)^2$ **b** $(y^2-3)^2$ **c** $(3a^2+4)^2$
d $(1-2x^2)^2$ **e** $(x^2+y^2)^2$ **f** $(x^2-a^2)^2$

6 Expand and simplify:

a $3x+1-(x+3)^2$ **b** $5x-2+(x-2)^2$
c $(x+2)(x-2)+(x+3)^2$ **d** $(x+2)(x-2)-(x+3)^2$
e $(3-2x)^2-(x-1)(x+2)$ **f** $(1-3x)^2+(x+2)(x-3)$
g $(2x+3)(2x-3)-(x+1)^2$ **h** $(4x+3)(x-2)-(2-x)^2$
i $(1-x)^2+(x+2)^2$ **j** $(1-x)^2-(x+2)^2$

G FURTHER EXPANSION

In this section we expand more complicated expressions by repeated use of the expansion laws.

Consider the expansion of $(a+b)(c+d+e)$.

Now $(a+b)(c+d+e)$
$= (a+b)c + (a+b)d + (a+b)e$
$= ac + bc + ad + bd + ae + be$

Compare: $\square(c+d+e)$
$= \square c + \square d + \square e$

Notice that there are 6 terms in this expansion and that each term within the first bracket is multiplied by each term in the second.

2 terms in the first bracket $\times$ 3 terms in the second bracket $\longrightarrow$ 6 terms in the expansion.

Example 16 **Self Tutor**

Expand and simplify: $(2x+3)(x^2+4x+5)$

$(2x+3)(x^2+4x+5)$
$= 2x^3 + 8x^2 + 10x$ {all terms of 2nd bracket $\times$ $2x$}
$\quad + 3x^2 + 12x + 15$ {all terms of 2nd bracket $\times$ 3}
$= 2x^3 + 11x^2 + 22x + 15$ {collecting like terms}

Example 17 — Self Tutor

Expand and simplify: $(x+2)^3$

$$\begin{aligned}(x+2)^3 &= (x+2)\times(x+2)^2\\ &= (x+2)(x^2+4x+4)\\ &= x^3+4x^2+4x \quad \{\text{all terms in 2nd bracket} \times x\}\\ &\quad\ +2x^2+8x+8 \quad \{\text{all terms in 2nd bracket} \times 2\}\\ &= x^3+6x^2+12x+8 \quad \{\text{collecting like terms}\}\end{aligned}$$

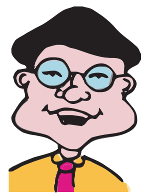
Example 18 — Self Tutor

Expand and simplify:

a $x(x+1)(x+2)$ **b** $(x+1)(x-2)(x+2)$

a
$$\begin{aligned}& x(x+1)(x+2)\\ &= (x^2+x)(x+2) \quad \{\text{all terms in first bracket} \times x\}\\ &= x^3+2x^2+x^2+2x \quad \{\text{expanding remaining factors}\}\\ &= x^3+3x^2+2x \quad \{\text{collecting like terms}\}\end{aligned}$$

b
$$\begin{aligned}& (x+1)(x-2)(x+2)\\ &= (x+1)(x^2-4) \quad \{\text{difference of two squares}\}\\ &= x^3-4x+x^2-4 \quad \{\text{expanding factors}\}\\ &= x^3+x^2-4x-4\end{aligned}$$



EXERCISE 3G

1 Expand and simplify:

a $(x+3)(x^2+x+2)$ **b** $(x+4)(x^2+x-2)$

c $(x+2)(x^2+x+1)$ **d** $(x+5)(x^2-x-1)$

e $(2x+1)(x^2+x+4)$ **f** $(3x-2)(x^2-x-3)$

g $(x+2)(2x^2-x+2)$ **h** $(2x-1)(3x^2-x+2)$

2 Expand and simplify:

a $(x+1)^3$ **b** $(x+3)^3$ **c** $(x-1)^3$

d $(x-3)^3$ **e** $(2x+1)^3$ **f** $(3x-2)^3$

3 Expand and simplify:

a $x(x+2)(x+3)$ **b** $x(x-4)(x+1)$ **c** $x(x-3)(x-2)$

d $2x(x+3)(x+1)$ **e** $2x(x-4)(1-x)$ **f** $-x(3+x)(2-x)$

g $-3x(2x-1)(x+2)$ **h** $x(1-3x)(2x+1)$ **i** $2x^2(x-1)^2$

4 Expand and simplify:

a $(x+3)(x+2)(x+1)$
b $(x-2)(x-1)(x+4)$
c $(x-4)(x-1)(x-3)$
d $(2x-1)(x+2)(x-1)$
e $(3x+2)(x+1)(x+3)$
f $(2x+1)(2x-1)(x+4)$
g $(1-x)(3x+2)(x-2)$
h $(x-3)(1-x)(3x+2)$

H THE BINOMIAL EXPANSION

Consider $(a+b)^n$. We note that:

- $a+b$ is called a **binomial** as it contains two terms
- any expression of the form $(a+b)^n$ is called a **power of a binomial**
- the **binomial expansion** of $(a+b)^n$ is obtained by writing the expression without brackets.

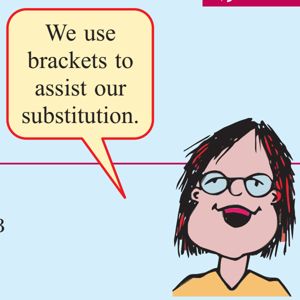
Now
$$\begin{aligned}(a+b)^3 &= (a+b)^2(a+b)\\ &= (a^2+2ab+b^2)(a+b)\\ &= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3\\ &= a^3+3a^2b+3ab^2+b^3\end{aligned}$$

So, the **binomial expansion** of $\boldsymbol{(a+b)^3 = a^3+3a^2b+3ab^2+b^3}$.

Example 19 **Self Tutor**

Expand and simplify using the rule $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$:

a $(x+2)^3$
b $(2x-1)^3$



a We substitute $a = x$ and $b = 2$

$$\begin{aligned}\therefore\quad (x+2)^3 &= x^3+3\times x^2\times 2+3\times x\times 2^2+2^3\\ &= x^3+6x^2+12x+8\end{aligned}$$

b We substitute $a=(2x)$ and $b=(-1)$

$$\begin{aligned}\therefore\quad (2x-1)^3 &= (2x)^3+3\times(2x)^2\times(-1)+3\times(2x)\times(-1)^2+(-1)^3\\ &= 8x^3-12x^2+6x-1\end{aligned}$$

EXERCISE 3H

1 Use the binomial expansion for $(a+b)^3$ to expand and simplify:

a $(x+1)^3$
b $(a+3)^3$
c $(x+5)^3$
d $(x-1)^3$
e $(x-2)^3$
f $(x-3)^3$
g $(3+a)^3$
h $(3x+2)^3$
i $(2x+3y)^3$

2 Copy and complete the argument $(a+b)^4 = (a+b)(a+b)^3$
$= (a+b)(a^3+3a^2b+3ab^2+b^3)$
$\vdots$

3 Use the binomial expansion $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$ to expand and simplify:

a $(x+1)^4$ **b** $(y+2)^4$ **c** $(3+a)^4$ **d** $(b+4)^4$

e $(x-1)^4$ **f** $(y-2)^4$ **g** $(3-a)^4$ **h** $(b-4)^4$

4 Find the binomial expansion of $(a+b)^5$ by considering $(a+b)(a+b)^4$.
Hence, write down the binomial expansion for $(a-b)^5$.

REVIEW SET 3A

1 Expand and simplify:

a $4x \times -8$ **b** $5x \times 2x^2$ **c** $-4x \times -6x$

d $3x \times x - 2x^2$ **e** $4a \times c + 3c \times a$ **f** $2x^2 \times x - 3x \times x^2$

2 Expand and simplify:

a $-3(x+6)$ **b** $2x(x^2-4)$

c $2(x-5)+3(2-x)$ **d** $3(1-2x)-(x-4)$

e $2x-3x(x-2)$ **f** $x(2x+1)-2x(1-x)$

g $x^2(x+1)-x(1-x^2)$ **h** $9(a+b)-a(4-b)$

3 Expand and simplify:

a $(3x+2)(x-2)$ **b** $(2x-1)^2$ **c** $(4x+1)(4x-1)$

d $(5-x)^2$ **e** $(3x-7)(2x-5)$ **f** $x(x+2)(x-2)$

g $(3x+5)^2$ **h** $-(x-2)^2$ **i** $-2x(x-1)^2$

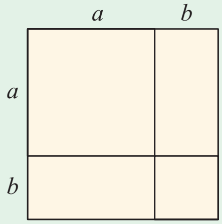
4 Expand and simplify:

a $5+2x-(x+3)^2$ **b** $(x+2)^3$

c $(3x-2)(x^2+2x+7)$ **d** $(x-1)(x-2)(x-3)$

e $x(x+1)^3$ **f** $(x^2+1)(x-1)(x+1)$

5



Explain how to use the given figure to show that $(a+b)^2 = a^2+2ab+b^2$.

REVIEW SET 3B

1 Expand and simplify:

a $3x \times -2x^2$ **b** $2x^2 \times -3x$ **c** $-5x \times -8x$
d $(2x)^2$ **e** $(-3x^2)^2$ **f** $4x \times -x^2$

2 Expand and simplify:

a $-7(2x-5)$ **b** $2(x-3)+3(2-x)$
c $-x(3-4x)-2x(x+1)$ **d** $2(3x+1)-5(1-2x)$
e $3x(x^2+1)-2x^2(3-x)$ **f** $3(2a+b)-5(b-2a)$

3 Expand and simplify:

a $(2x+5)(x-3)$ **b** $(3x-2)^2$ **c** $(2x+3)(2x-3)$
d $(5x-1)(x-2)$ **e** $(2x-3)^2$ **f** $(1-5x)(1+5x)$
g $(5-2x)^2$ **h** $-(x+2)^2$ **i** $-3x(1-x)^2$

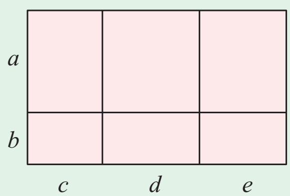
4 Expand and simplify:

a $(2x+1)^2-(x-2)(3-x)$ **b** $(x^2-4x+3)(2x-1)$
c $(x+3)^3$ **d** $(x+1)(x-2)(x+5)$
e $2x(x-1)^3$ **f** $(4-x^2)(x+2)(x-2)$

5 Use the binomial expansion $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$ to expand and simplify:

a $(2x+1)^4$ **b** $(x-3)^4$

6



What algebraic fact can you derive by considering the area of the given figure in two different ways?